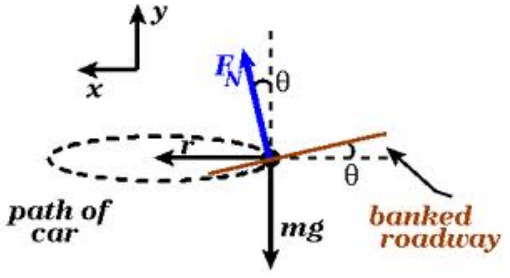
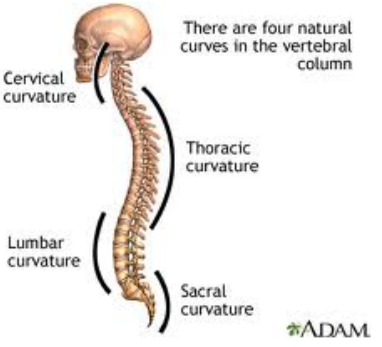
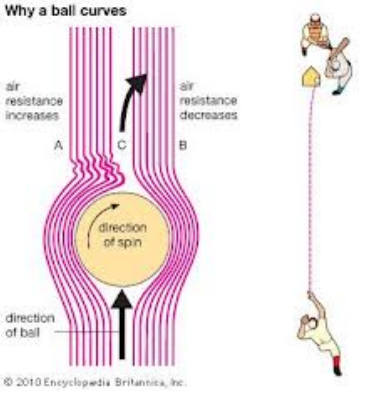
Normal Curves

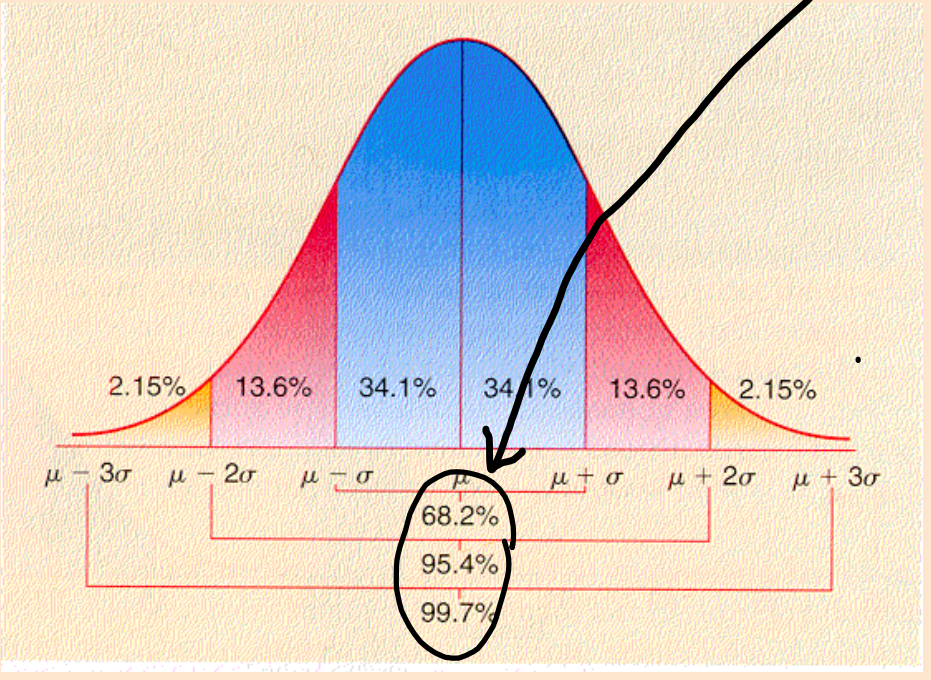






The Normal Curve

Empirical Rule



What is Normal?

What is a normal heart rate?

Parakeet (aka budgerigar) budgie
- Heart Rate 250 to 600 beats per minute
- Body Temp 105°F to 108.6°F

Hummingbird
- Heart Rate 800+ beats per minute
- Body Temp 108°F to 111.2°F

Canine
- Heart Rate 80 to 150 beats per minute
- Body Temp 100°F to 102°F

Feline
- Heart Rate 110 to 140 beats per minute
- Body Temp 100.5°F to 102.5°F

Human
- Heart Rate 60 to 100 beats per minute
- Body Temp 97.6°F to 98.8°F

Why do mammals have a body temperature around 96°F?

What's normal for poop?

Defecation duration is constant across many animal species – around 12 seconds (plus or minus 7 seconds) – even though the volume varies greatly.

Assuming a bell curve distribution, 66 percent of animals take between 5 and 19 seconds to defecate.

It's a surprisingly small range, given that elephant feces have a volume of 20 liters, nearly a thousand times more than a dog's, at 10 milliliters.

*Colorectal surgeon **Daniel Chu**, and undergraduates, Candice Kaminski and Morgan LaMarca as reported on BBC.com 2017*

How did they come up with the *normal range*?

By Sampling

Let's do the class heart rate..

go to class heart rate

- If you take a large enough random sample, you will get a *Normal Distribution*.

- Plotted on a dot plot, or histogram with small enough classes, you will see a normal curve.

- Used for continuous numerical data
e.g. temperature, volume, time, mg/dl, height

- Not for discrete data

Properties of the normal curve

- Mean, median, and mode are equal and located at the center of the curve.
- Symmetric about the mean
- Continuous
- Never touches the x-axis
- Area under the curve = 1.00 or 100%

Go to Normal.ftm

What is the y-axis on the graph of a normal curve?

What is the x-axis on the graph of a normal curve?

What is the area under the normal distribution curve?

Go to Building Normal Distribution

What use is the Normal Distribution?

Calculating probability.

Determining if a sample or population is "normal".

Confidence Intervals.

Equation for the normal curve:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Calculating probability from normal distribution:

Use integration:

$$p(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

or...

Use a z-score table

Z Table

Entries in the body of the table represents areas under the curve between $-\infty$ and z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	**0.5000**	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	**0.9495**	**0.9505**	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	**0.9750**	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

C:\DATA\StatPrimer\z-table.wpd (12/11/03)

http://davidmlane.com/hyperstat/z_table.html

Use your calculator:

DISTR 2

normalcdf(z_1, z_2, μ, σ)

The standard normal curve has μ = 0 and σ = 1
You don't have to enter μ = 0 and σ = 1.

Find the area under the standard normal curve between $z_1 = 0$ and $z_2 = 1.66$

Find the area under the standard normal curve between $z_1 = 0.5$ and $z_2 = 1.5$

Find the area under the std normal curve for values of z between these values:

0 and 0.75

0 and -2.07

more than 1.10

less than -0.48

-1.03 and 2.47

less than -1.20 and more than 1.20

Bring Pennies!

Remember: **area = probability**

Using the standard normal curve, find these probabilities:

$p(z > 1.69) =$

$p(z > -1.25) =$

Remember: **area = probability**

Using the standard normal curve, find these probabilities:

$p(-1.00 < z < 1.00) =$

$p(-1.00 < z < 2.00) =$

Remember: **area = probability**

Using the standard normal curve, find these probabilities:

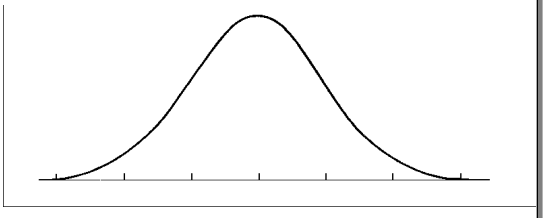
$p(z < 2.00) =$

$p(z > -1.50) =$

Remember the standard normal curve has $\mu = 0$ and $\sigma = 1$

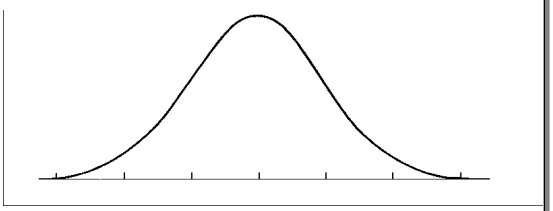
e.g. 1

An unspecified test had a mean of 25 with a s.d. of 4. Find the percentage of tests that scored less than 30.

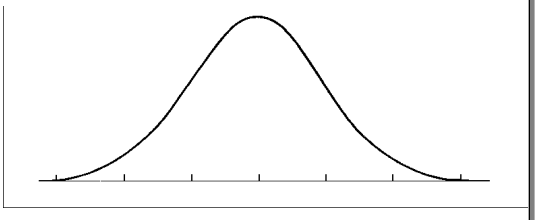


e.g.2

Each month, a typical household in the USA generates about 28 pounds of newspapers for recycling. If the s.d. is 2 pounds, find the percent of households that generate more than 30.2 pounds.



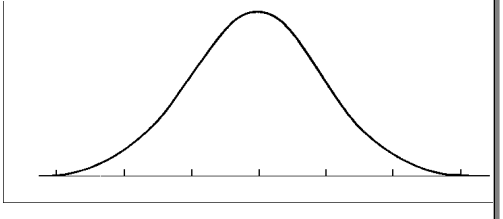
e.g. 3
An automobile gets an average of 22 miles per gallon (mpg) in the city, with s.d. 3mpg. Find the probability that a randomly selected car will get greater than 26 mpg in the city.



e.g.4

To qualify for entry into the Police Academy an applicant must score in the top 10% in an entrance exam.

If the scores are normally distributed with a mean of 200 and s.d. 20, what is the cut off score?

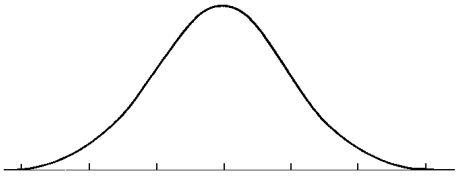


http://www.uvm.edu/~dhowell/StatPages/More_Stuff/normalcdf.html

e.g.5
A medical researcher wants to test her new heart disease drug on a large sample of fairly "normal" people. So she decides to select from the middle 60% of the population.

In the population generally the mean systolic blood pressure is 120 with s.d. 8.

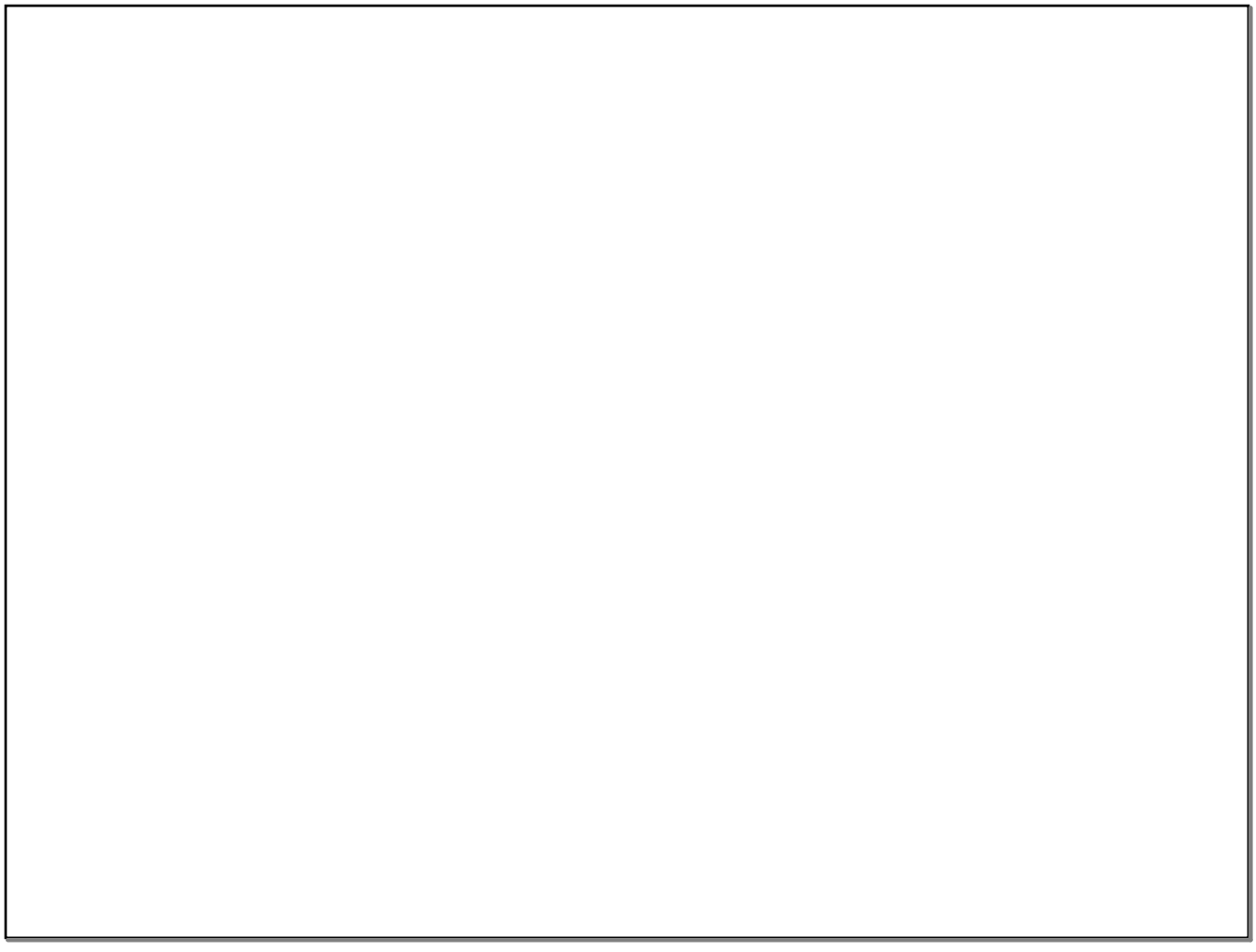
Find the upper and lower blood pressure readings that would qualify people for the study.



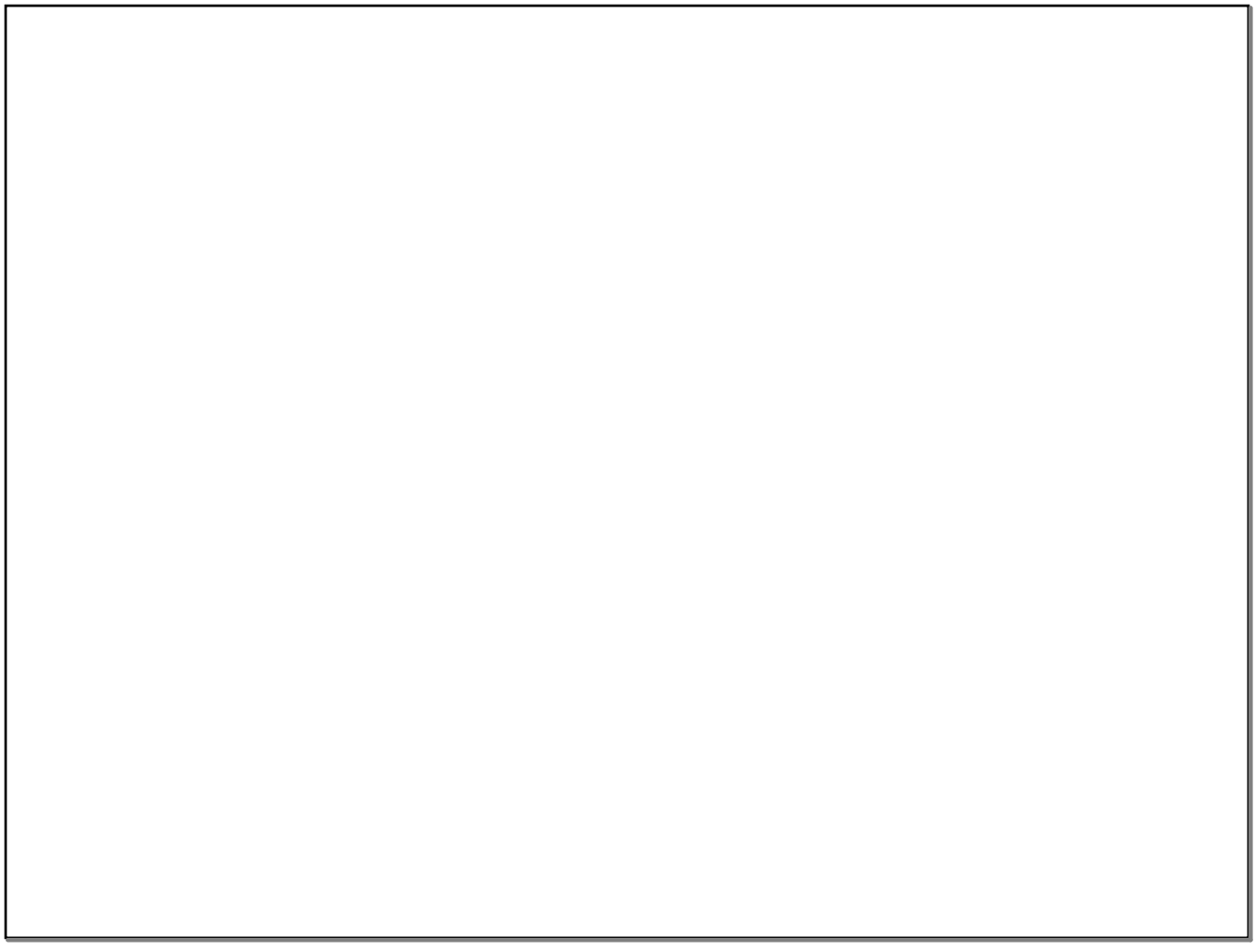
Your Turn...

Assignment time

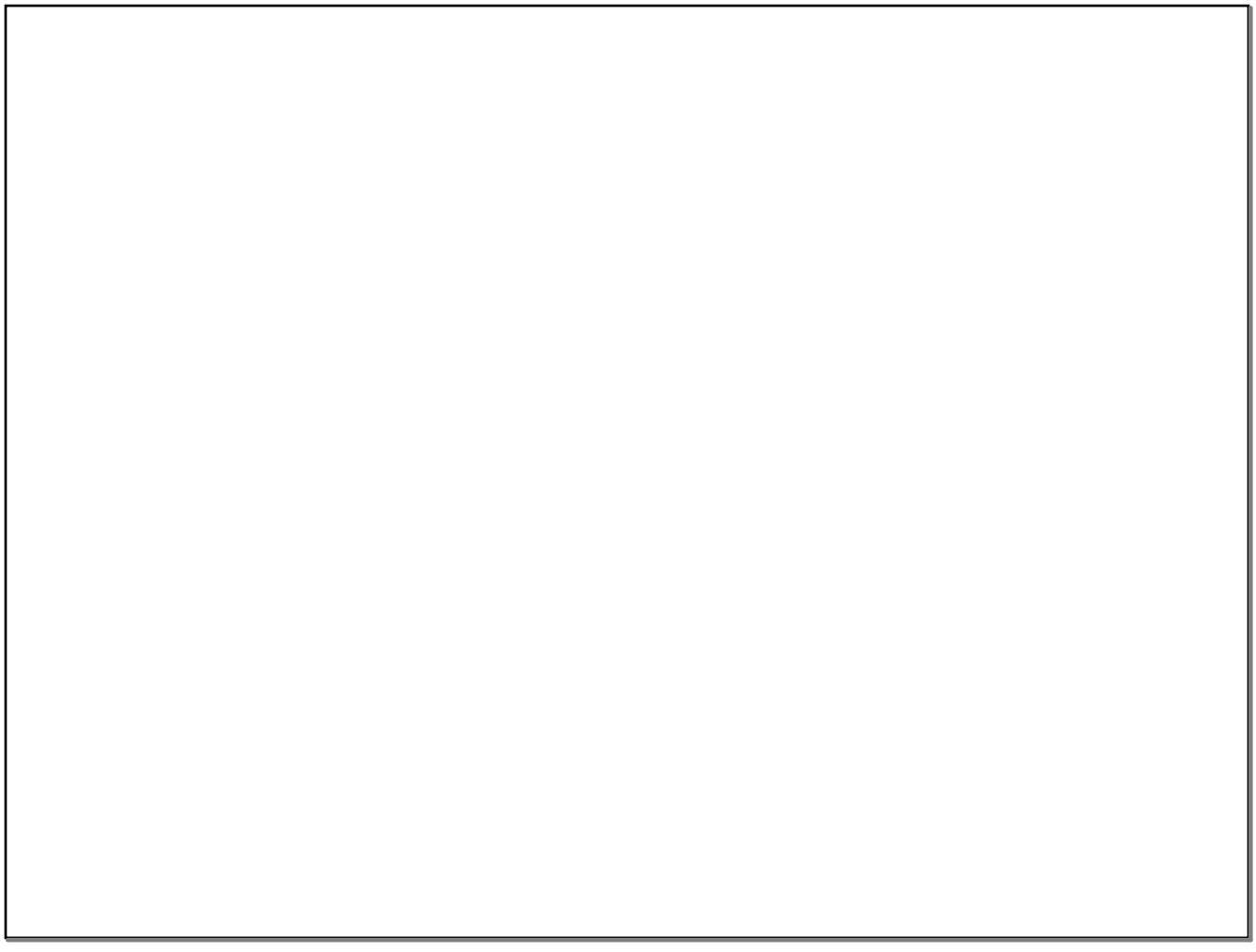
May 16, 2018



May 16, 2018



May 16, 2018



1. To screen employees for a proof reading job a publishing company gives new applicants a speed reading test. Only the top 15% get interviews. Assume a normal distribution with mean 600 words per minute and s.d. 100 words per minute, find the minimum reading speed needed to be accepted for an interview.

2. The quality control people at your factory tell you that your new product, the Wundafone, has a mean lifetime of 25 months, with s.d. of 5 months. If you have to offer a replacement guarantee, how many months should you allow if you don't want to replace more than 8% of your Wundafones?

3. The length of human pregnancies is normal with mean 266 days and s.d. 16 days. What is the probability that a randomly selected pregnancy will last less than 260 days?

4. Since the 1900s the magnitude of earthquakes in California that measure 0.1 or higher on the Richter Scale is approx. normal with mean 6.2 and s.d. 0.5.

a. What range of Richter Scale values represent the 20% most powerful earthquakes in CA?

b. Determine the range of Richter Scale values that make up the middle 85% of earthquake magnitudes.

5. IQ scores on the Standford-Binet intelligence tests are normally distributed with mean 100 and s.d. 16.

a. In order to qualify for Mensa, you must score in the top 2%. What IQ score is required to qualify for Mensa?

b. What range of IQ scores make up the middle 50% of the population?

fin

What is the z-score?

Calculating z scores:

$$z = \frac{x - \mu}{\sigma}$$